FTUAM-15-16, IFT-UAM/CSIC-15-058, TUM-HEP-997/15

One-loop renormalization of the electroweak chiral Lagrangian with a light Higgs

Feng-Kun Guo [1], Pedro Ruiz-Femenía [2], and Juan José Sanz-Cillero [3]
[1] *Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*
[2] *Physik Department T31, James-Franck-Strasse, Technische Universität München, D-85748 Garching, Germany and*
[3] *Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*

We consider the general chiral effective action which parametrizes the nonlinear realization of the spontaneous breaking of the electroweak symmetry with a light Higgs, and compute the one-loop ultraviolet divergences coming from Higgs and electroweak Goldstone fluctuations using the background field method. The renormalization of the divergences is carried out through operators of next-to-leading order in the chiral counting, i.e. of $\mathcal{O}(p^4)$. Being of the same order in power counting, the logarithmic corrections linked to these divergences can be as important as the tree-level contributions from the $\mathcal{O}(p^4)$ operators, and must be accounted for in the phenomenological analysis of experimental data. Deviations in the $\mathcal{O}(p^2)$ (leading-order) couplings with respect to the Standard Model values, *e.g.*, in the $h \to WW$ coupling, would generate contributions from the 1-loop chiral logarithms computed in this work to a vast variety of observables, which do not have a counterpart in the conventional electroweak effective theory with a linearly transforming Higgs complex doublet.

INTRODUCTION

The lack of experimental evidence of the new physics states predicted by many natural solutions to the electroweak (EW) symmetry breaking has brought an increasing interest in the analysis of possible deviations from the Standard Model (SM) using broader frameworks. Assuming there is a large energy gap between the EW scale $v = 246$ GeV and the new physics scale, a description of the EW symmetry breaking (EWSB) in general terms is provided by an effective field theory (EFT) built from the presently known particle content, including a light Higgs scalar, and based on the spontaneous symmetry breaking pattern of the SM: $G = SU(2)_L \times SU(2)_R$ breaks down to the custodial group $H = SU(2)_{L+R}$, where $SU(2)_L \times U(1)_Y \subset G$ is gauged and the three would-be Goldstone bosons π^a that arise from the spontaneous EWSB give mass to the $W^\pm, Z$ in the unitary gauge.

The existence of an approximate custodial symmetry guarantees that the ρ-parameter corrections are small, the latter being originated from radiative corrections or (custodial-symmetry breaking) operators that are subleading in the EFT power-counting [1]. Without loss of generality, the Higgs h will be taken to be a singlet under the full group G and the Goldstones are nonlinearly realized in our EFT approach [2]; linear models with a Higgs complex doublet Φ (e.g. the SM) are just a subset within the general class of nonlinear theories, as it is always possible to express Φ in terms of the singlet h and the nonlinearly realized Goldstones. Indeed, there is a vast variety of beyond-SM theories where the Higgs is a composite particle, typically a pseudo-Goldstone of some type, and shows the characteristic nonlinear interaction structure of this kind of particles (see the review [3]).

The framework described above shares many similarities with the low-energy limit of QCD, also ruled by the $SU(2)_L \times SU(2)_R$ chiral symmetry, and we can expect that well-known aspects from QCD are reproduced in the nonlinear EW chiral EFT. In particular, we are interested in this work in the relevant role of the EW chiral logarithms arising from radiative corrections. We know from their QCD analogues that such contributions to one-loop amplitudes are in many cases as important as the tree-level contributions from higher dimension operators, due to the nonlinear structure of the Goldstone interactions (that is the case for instance in $\pi\pi$ scattering in the scalar-isoscalar channel [4]).

Motivated by this fact, in this letter we study the radiative corrections coming from scalar boson loops (Higgs and EW Goldstones) within the framework of a nonlinear EW chiral Lagrangian including a light Higgs (ECLh). Ref. [5] computed the ultraviolet (UV) divergence at next-to-leading order (NLO) in the linear Higgs EFT. The present work complements that analysis and provides the UV divergences and the renormalization for the nonlinear case at NLO in the low-energy chiral counting, i.e. at $\mathcal{O}(p^4)$, where p denotes low-energy scales, either light-particle masses or momenta. The ECLh contains all the SM particles and it is expected to describe their interaction at energies much below the cut-off of the EFT, Λ_{ECLh}, given by either the scale at which one encounters a new state or the energy where loop corrections turn too large to validate a perturbative expansion, naively $4\pi v \approx 3$ TeV.

Based on dimensional analysis it is possible to organize a low-energy expansion of the amplitudes in powers of low-energy scales p [6, 7] and implement a chiral power counting in the nonlinear EFT Lagrangian [8–10].

Renormalization is then carried out order-by-order in the low-energy expansion [4, 11–13].

The one-loop effective action shows a series of UV divergences of higher dimension that require the inclusion of new operators in the Lagrangian, which are NLO in the chiral counting. While the precise value of the NLO couplings depends on the underlying physics, their running is fully determined by the leading order (LO) Lagrangian and its symmetry structure. Theoretical predictions that incorporate the latter operators must therefore account also for the logarithmic contributions in order to reach the percent accuracy level in the data analyses of next runs at present and future colliders [14]. Unless assumptions about the physics underlying the ECLh are taken, such phenomenological studies must account in full generality for the tree-level contributions from NLO operators [10, 15], the running and the associated NLO logarithms (that are provided in this letter), and the NLO loop finite pieces [8, 16–19], which vary from one observable to another.

LOW-ENERGY EFFECTIVE THEORY

At low energies the amplitudes can be then organized in terms of a chiral expansion in powers of the low-energy scales p. Analyzing the contributions from the effective operators to the amplitudes it is possible to establish a consistent (chiral) power counting for the ECLh Lagrangian [8–10], which is organized as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots , \tag{1}$$

where $\mathcal{L}_n$ contains terms that scale as $\mathcal{O}(p^n)$. The LO Lagrangian is given by [10, 15, 20, 21],

$$\begin{aligned}\mathcal{L}_2 = & \frac{v^2}{4}\mathcal{F}_C\langle\, u_\mu u^\mu \,\rangle + \frac{1}{2}(\partial_\mu h)^2 - v^2\, V \\ & +\mathcal{L}_{YM} + i\bar\psi \, \not{D}\psi \, - \, v^2\langle\, J_S \,\rangle ,\end{aligned} \tag{2}$$

where $\langle \ldots \rangle$ stands for the trace of the 2×2 EW tensors, $\mathcal{L}_{YM}$ is the Yang–Mills Lagrangian for the gauge fields, D is the gauge covariant derivative acting on the fermions, and J_S denotes the Yukawa coupling of the fermions to the Higgs and Goldstone fields defined below. The factors of v in the normalization of some terms are introduced for later convenience. $\mathcal{F}_C$, V and J_S are functionals of $x = h/v$, and have Taylor expansions

$$\begin{aligned}&\mathcal{F}_C[x] = 1 + 2ax + bx^2 + \ldots , \qquad J_S[x] = \sum_n J_S^{(n)} x^n/n! \, , \\ &V[x] = m_h^2\left(\frac{1}{2}x^2 + \frac{1}{2}d_3x^3 + \frac{1}{8}d_4x^4 \, + \ldots\right) ,\end{aligned} \tag{3}$$

given in terms of the constants a, b, m_h, etc. [10, 15, 20, 21], with $J_S^{(n)}$ the n-th derivative with respect to h/v. In the nonlinear realization of the spontaneous EWSB the Goldstones are parameterized through the coordinates (u_L, u_R) of the $SU(2)_L\times SU(2)_R/SU(2)_{L+R}$ coset space [2], with the unitary matrices $u_{L,R}$ being functions of the Goldstone fields π^a which enter through the building blocks

$$\begin{aligned} u_\mu &= iu_R^\dagger(\partial_\mu - ir_\mu)u_R - iu_L^\dagger(\partial_\mu - i\ell_\mu)u_L \, , \\ \Gamma_\mu &= \frac{1}{2}u_R^\dagger(\partial_\mu - ir_\mu)u_R + \frac{1}{2}u_L^\dagger(\partial_\mu - i\ell_\mu)u_L \, , \\ f_\pm^{\mu\nu} &= u_L^\dagger \ell^{\mu\nu} u_L \pm u_R^\dagger r^{\mu\nu} u_R \, , \end{aligned} \tag{4}$$

where $r_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$, and the left-hand counter part $\ell_{\mu\nu}$ is defined analogously. The tensor J_S is defined as

$$\begin{aligned} J_S &= J_{YRL} + J_{YRL}^\dagger , \\ J_{YRL} &= -\frac{1}{\sqrt{2}v} u_R^\dagger \, \hat{Y} \; \psi_R^\alpha \bar\psi_L^\alpha u_L \, , \end{aligned} \tag{5}$$

where $\psi_{R,L} = \frac{1}{2}(1\pm\gamma^5)\psi$ and $\psi = (t,b)^T$ is the top-bottom SM $SU(2)$ doublet. The inclusion of additional doublets is straightforward and will be discussed later. The summation over the Dirac index α in $\psi_R^\alpha\bar\psi_L^\alpha$ is assumed and its tensor structure under G is left implicit. The 2×2 matrix $\hat{Y}[h/v]$ is a spurion auxiliary field, functional of h/v, which incorporates the fermionic Yukawa coupling, allowing the inclusion of explicit custodial symmetry-breaking terms [9].

The low-energy chiral counting of the building blocks is provided by the scaling $\{\partial_\mu, r_\mu, \ell_\mu, m_{h,W,Z,\psi}\} \sim \mathcal{O}(p)$, $\{\pi^a/v,\, u_{L,R},\, h/v\} \sim \mathcal{O}(p^0)$, and $\psi/v \sim O(p^{1/2})$ [6, 7]. Accordingly, covariant derivatives must scale as the ordinary ones and $g, g', \hat{Y} \sim \mathcal{O}(p/v)$. Since this implies $\mathcal{F}_C \sim \mathcal{O}(p^0)$, $u_\mu \sim \mathcal{O}(p)$ and $f_\pm^{\mu\nu}$, J_{YRL}, J_S, $V \sim \mathcal{O}(p^2)$, the LO Lagrangian in Eq. (2) is $\mathcal{O}(p^2)$ and the one-loop corrections are formally $\mathcal{O}(p^4)$ [6–10].

The transformations of the building blocks under G are given by

$$\begin{aligned} & h \to h \, , \qquad Y \to g_R Y g_R^\dagger \, , \qquad u_{R/L} \to g_{R/L}\, u_{R/L}\, g_h^\dagger \, , \\ & r_\mu \to g_R r_\mu g_R^\dagger + ig_R\partial_\mu g_R^\dagger \, , \qquad \ell_\mu \to g_L \ell_\mu g_L^\dagger + ig_L\partial_\mu g_L^\dagger \, , \\ & \mathcal{O} \to g_h\, \mathcal{O}\, g_h^\dagger \quad \text{for} \quad \mathcal{O} \in \{\, u_\mu,\, J_{YRL},\, J_S \, ,\, f_\pm^{\mu\nu} \,\} , \end{aligned} \tag{6}$$

where $g_{R/L} \in SU(2)_{R/L}$, and $g_h \in SU(2)_{L+R}$ is the compensating transformation of the Goldstone representatives in the CCWZ formalism [2].

The SM is recovered by setting $\mathcal{F}_C = (1 + h/v)^2$, $V = \frac{1}{4}\lambda v^2[(1 + h/v)^2 - 1]^2$ and $\hat{Y} = (1 + h/v)\,(y_t P_+ + y_b P_-)$, defined in terms of the SM Yukawa coupling constants y_q and the projectors $P_\pm = (1\pm\sigma^3)/2$ (σ^a are the Pauli matrices). Other SM fermion doublets and the flavor symmetry breaking between generations can be incorporated by adding in J_{YRL} an additional family index in the fermion fields, ψ^A, and promoting $\hat{Y}$ to a tensor $\hat{Y}^{AB}$ in the generation space [22].

In our analysis, ℓ_μ, r_μ, $\hat{Y}$ are spurion auxiliary background fields that keep the invariance of the ECLh action under G. When evaluating physical matrix elements,

custodial symmetry is then broken in the same way as in the SM, keeping only the gauge invariance under the subgroup $SU(2)_L \times U(1)_Y \subset G$ [10, 11, 15, 20]:

$$\ell_\mu = -\frac{g}{2} W_\mu^a \sigma^a \,, \quad r_\mu = -\frac{g'}{2} B_\mu \sigma^3 \,, \\ \hat{Y} = \hat{y}_t P_+ \, + \, \hat{y}_b P_- \,, \tag{7}$$

where $\hat{y}_{t,b}$ must be also understood as functionals of h/v. Notice that the Yukawa operators are still invariant under global G transformations if $\hat{y}_t = \hat{y}_b$.

EFFECTIVE ACTION AT ONE LOOP

Our aim is to compute the one-loop UV divergences of the effective action by means of the background field method [23]. We choose for the G/H coset representatives $u_L = u_R^\dagger$ [24] and perform fluctuations of the scalar fields (Higgs and Goldstones) around the classical background fields $\bar{h}$ and $\bar{u}_{L,R}$, respectively, in a way analogous to Ref. [25]:

$$u_{R,L} = \bar{u}_{R,L} \, \exp\{\pm i \mathcal{F}_C^{-1/2} \, \Delta/(2v)\} \,, \quad h = \bar{h} \, + \, \epsilon \,, \tag{8}$$

with $\Delta = \Delta^a \sigma^a$. Without any loss of generality we have introduced the factor $\mathcal{F}_C^{-1/2}$ in the exponent for later convenience; it will allow us to write down the second-order fluctuation of the action in the canonical form [23].

To obtain the one-loop effective action within the background field method one retains the quantum fluctuations $\vec{\eta}^T = (\Delta^a, \epsilon)$ up to quadratic order [23],

$$\mathcal{L}_2 = \mathcal{L}_2^{\mathcal{O}(\eta^0)} \, + \, \mathcal{L}_2^{\mathcal{O}(\eta^1)} \, + \, \mathcal{L}_2^{\mathcal{O}(\eta^2)} \, + \, \mathcal{O}(\eta^3) \,, \tag{9}$$

where $\mathcal{L}_2^{\mathcal{O}(\eta^0)} = \mathcal{L}_2[\bar{u}_{L,R}, \bar{h}]$. The tree-level effective action is equal to the action evaluated at the classical solution, $\int \mathrm{d^d x} \, \mathcal{L}^{\mathcal{O}(\eta^0)}$, where $\int \mathrm{d^d x} \, \mathcal{L}_2^{\mathcal{O}(\eta^0)}$ denotes the LO contribution in the chiral expansion. The background field configurations $\bar{h}$ and $\bar{u}_{L,R}$ correspond to the solutions of the classical equations of motion (EoM), defined by the vanishing of the linear term $\mathcal{L}_2^{\mathcal{O}(\eta)}$ for arbitrary $\vec{\eta}$. They read

$$\nabla^\mu u_\mu = -2 J_P / \mathcal{F}_C - u_\mu \partial^\mu (\ln \mathcal{F}_C) \,, \\ \partial^2 h / v = \frac{1}{4} \mathcal{F}_C' \, \langle \, u_\mu u^\mu \, \rangle - V' - \langle \, J_S' \, \rangle \,, \tag{10}$$

with $J_P = i(J_{YRL} - J_{YRL}^\dagger)$ and the covariant derivative $\nabla_\mu \cdot = \partial_\mu + [\Gamma_\mu, \cdot]$. Here and in the following, we abuse of the notation by writing the background fields $\bar{u}_\mu$ and $\bar{h}$ as u_μ and h for conciseness.

The quadratic fluctuation $\mathcal{L}_2^{\mathcal{O}(\eta^2)}$ reads

$$\mathcal{L}^{\mathcal{O}(\Delta^2)} = -\frac{1}{4} \langle \, \Delta \nabla^2 \Delta \, \rangle \, + \, \frac{1}{16} \langle \, [u_\mu, \Delta] \, [u^\mu, \Delta] \, \rangle \\ + \left[\frac{\mathcal{F}_C^{-\frac{1}{2}} \mathcal{K}}{8} \left(\frac{\partial^2 h}{v} \right) + \frac{\Omega}{16} \left(\frac{\partial_\mu h}{v} \right)^2 \right] \langle \, \Delta^2 \, \rangle + \frac{1}{2\mathcal{F}_C} \langle \, \Delta^2 J_S \, \rangle , \\ \mathcal{L}^{\mathcal{O}(\epsilon^2)} = -\frac{1}{2} \epsilon \left[\partial^2 - \frac{1}{4} \mathcal{F}_C'' \langle \, u_\mu u^\mu \, \rangle + V'' + \langle \, J_S'' \, \rangle \right] \epsilon \,, \\ \mathcal{L}^{\mathcal{O}(\epsilon\Delta)} = \, -\frac{1}{2} \epsilon \mathcal{F}_C' \, \langle \, u_\mu \nabla^\mu (\mathcal{F}_C^{-\frac{1}{2}} \Delta) \, \rangle + \mathcal{F}_C^{-\frac{1}{2}} \epsilon \langle \, \Delta J_P' \, \rangle \,, \tag{11}$$

in terms of $\mathcal{K} = \mathcal{F}_C^{-1/2} \mathcal{F}_C'$ and $\Omega = 2\mathcal{F}_C''/\mathcal{F}_C - (\mathcal{F}_C'/\mathcal{F}_C)^2$. Through a proper definition of the differential operator $d_\mu \vec{\eta} = \partial_\mu \vec{\eta} + Y_\mu \vec{\eta}$, one can rewrite $\mathcal{L}_2^{\mathcal{O}(\eta^2)}$ in the canonical form

$$\mathcal{L}_2^{\mathcal{O}(\eta^2)} = -\frac{1}{2} \vec{\eta}^T \, (d_\mu d^\mu + \Lambda) \vec{\eta} \tag{12}$$

where d_μ and Λ depend on h, $u_{L,R}$ and on the gauge boson and fermion fields.

The $\mathcal{L}_2^{\mathcal{O}(\eta^2)}$ term in the generating functional is just a Gaussian integral over the quantum fluctuations $\vec{\eta}$, which can be performed with standard methods to provide the 1-loop effective action [23, 26]:

$$S^{1\ell} = \frac{i}{2} \mathrm{tr} \, \log \left(d_\mu d^\mu + \Lambda \right) , \tag{13}$$

where tr stands for the full trace of the operator, also in coordinate space. One can then extract the residue of the $1/(d-4)$ pole in dimensional regularization using the Schwinger–DeWitt proper-time representation of the operator in Eq. (13) and the heat-kernel expansion [23]:

$$S^{1\ell} = -\lambda \int \mathrm{d^d x} \; \mathrm{Tr} \left\{ \frac{1}{12} Y_{\mu\nu} Y^{\mu\nu} + \frac{1}{2} \Lambda^2 \right\} + \text{finite} \\ = -\lambda \int \mathrm{d^d x} \; \sum_k \Gamma_k \, \mathcal{O}_k \, + \, \text{finite} \,, \tag{14}$$

with $\lambda = [16\pi^2(d-4)]^{-1} \mu^{d-4}$. The divergence is determined by the non-derivative quadratic fluctuation Λ and the differential operator d_μ through $Y_{\mu\nu} = [d_\mu, d_\nu] = \partial_\mu Y_\nu - \partial_\nu Y_\mu + [Y_\mu, Y_\nu]$, and we note that both Λ and $Y_{\mu\nu}$ are $\mathcal{O}(p^2)$ in the chiral counting. In Eq. (14) Tr refers to the trace over the 4×4 operators that act on the fluctuation vector $\vec{\eta}^T = (\Delta_i, \epsilon)$. The basis of local operators $\mathcal{O}_k$ that covers the space of one-loop divergences contains purely bosonic terms (given in Table I) and operators including fermions (discussed later in Eq. (16)). For the UV-divergent part of the effective action we have a chiral expansion in powers of p similar to that for the EFT Lagrangian in (1): $\mathcal{L}^{1\ell,\infty} = \mathcal{L}_2^{1\ell,\infty} \, + \, \mathcal{L}_4^{1\ell,\infty} \, + \, ...$

The UV divergences with the structure of the $\mathcal{L}_2$ op-

c_k	Operator $\mathcal{O}_k$	Γ_k	$\Gamma_{k,0}$
c_1	$\frac{1}{4}\langle f_+^{\mu\nu} f_{+\,\mu\nu} - f_-^{\mu\nu} f_{-\,\mu\nu}\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
(c_2-c_3)	$\frac{i}{2}\langle f_+^{\mu\nu}[u_\mu, u_\nu]\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
c_4	$\langle u_\mu u_\nu\rangle \langle u^\mu u^\nu\rangle$	$\frac{1}{96}(\mathcal{K}^2-4)^2$	$\frac{1}{6}(1-a^2)^2$
c_5	$\langle u_\mu u^\mu\rangle^2$	$\frac{1}{192}(\mathcal{K}^2-4)^2 + \frac{1}{128}\mathcal{F}_C^2\Omega^2$	$\frac{1}{8}(a^2-b)^2 + \frac{1}{12}(1-a^2)^2$
c_6	$\frac{1}{v^2}(\partial_\mu h)(\partial^\mu h)\langle u_\nu u^\nu\rangle$	$\frac{1}{16}\Omega(\mathcal{K}^2-4) - \frac{1}{96}\mathcal{F}_C\Omega^2$	$-\frac{1}{6}(a^2-b)(7a^2-b-6)$
c_7	$\frac{1}{v^2}(\partial_\mu h)(\partial_\nu h)\langle u^\mu u^\nu\rangle$	$\frac{1}{24}\mathcal{F}_C\Omega^2$	$\frac{2}{3}(a^2-b)^2$
c_8	$\frac{1}{v^4}(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	$\frac{3}{32}\Omega^2$	$\frac{3}{2}(a^2-b)^2$
c_9	$\frac{(\partial_\mu h)}{v}\langle f_-^{\mu\nu} u_\nu\rangle$	$\frac{1}{24}\mathcal{F}_C'\Omega$	$-\frac{1}{3}a(a^2-b)$
c_{10}	$\frac{1}{2}\langle f_+^{\mu\nu} f_{+\,\mu\nu} + f_-^{\mu\nu} f_{-\,\mu\nu}\rangle$	$-\frac{1}{48}(\mathcal{K}^2+4)$	$-\frac{1}{12}(1+a^2)$

TABLE I. Purely bosonic operators needed for the renormalization of the NLO effective Lagrangian $\mathcal{L}_4$. In the last column, we provide the first term $\Gamma_{k,0}$ in the expansion of the Γ_k in powers of (h/v) by using $\mathcal{F}_C = 1 + 2ah/v + bh^2/v^2 + \mathcal{O}(h^3)$ (no further term is needed). The first five operators $\mathcal{O}_i$ have the structure of the respective a_i Longhitano operator [11, 12] (with $i = 1...5$). In addition, $c_6 = \mathcal{F}_{D7}$, $c_7 = \mathcal{F}_{D8}$ and $c_8 = \mathcal{F}_{D11}$ in the notation of Ref. [10]. The last operator of the list, $\mathcal{O}_{10} = 2\langle r_{\mu\nu} r^{\mu\nu} + \ell_{\mu\nu}\ell^{\mu\nu}\rangle$, only depends on the EW field strength tensors and its coefficient is labeled as $c_{10} = H_1$ in the notation of Ref. [4].

erators in Eq. (2) are given by

$$\mathcal{L}_2^{1\ell,\infty} = -\lambda\Bigg\{ \frac{1}{8}\left[\frac{\mathcal{F}_C' V'}{\mathcal{F}_C}(4-\mathcal{K}^2) - \mathcal{F}_C\Omega V''\right]\langle u_\mu u^\mu\rangle - \frac{3\mathcal{F}_C' V'\Omega}{8\mathcal{F}_C}\left(\frac{\partial_\mu h}{v}\right)^2 + \left[\frac{1}{2}(V'')^2 + \frac{3\mathcal{K}^2}{8\mathcal{F}_C}(V')^2\right] + \left(V''\langle J_S''\rangle - \frac{3\mathcal{F}_C' V'}{2\mathcal{F}_C}\langle \Gamma_S\rangle\right)\Bigg\}, \tag{15}$$

where we define $\Gamma_S = \mathcal{F}_C^{-1}(J_S - \mathcal{F}_C' J_S'/2)$. These UV divergences are cancelled out through the renormalization of the various parts of $\mathcal{L}_2$: the couplings in the $\mathcal{F}_C$ term (1st line); the Higgs kinetic term (1st term in 2nd line), which requires a NLO Higgs field redefinition; the coefficients of the Higgs potential, e.g. the Higgs mass (2nd bracket in 2nd line); the Yukawa term couplings in Y (3rd line).

The $\mathcal{O}(p^4)$ divergences $\mathcal{L}_4^{1\ell,\infty}$ are split here into two types, according to whether they include fermion fields or not. The purely bosonic $\mathcal{O}(p^4)$ divergences $\mathcal{L}_4^{1\ell,\infty}|_{\rm bos}$ are summarized in table I, where we provide the factors $\Gamma_k = \Gamma_k[h/v]$ corresponding to each $\mathcal{O}(p^4)$ operator $\mathcal{O}_k$ in the effective action.

The structure of $\mathcal{O}(p^4)$ UV divergences with fermion field operators is slightly more involved:

$$\mathcal{L}_4^{1\ell,\infty}|_{\rm ferm} = -\lambda\Bigg\{ \langle \left(\frac{\mathcal{K}^2}{4} - 1\right)\Gamma_S - \frac{\mathcal{F}_C\Omega}{8}J_S''\rangle\langle u^\mu u_\mu\rangle + \frac{3}{4}\Omega\langle\Gamma_S\rangle\left(\frac{\partial_\mu h}{v}\right)^2 + \frac{1}{2}\Omega\langle\Gamma_P u^\mu\rangle\left(\frac{\partial_\mu h}{v}\right) + \frac{1}{2}\langle J_S''\rangle^2 + \frac{3}{2}\langle\Gamma_S\rangle^2 + \frac{1}{\mathcal{F}_C}\left(2\langle\Gamma_P^2\rangle - \langle\Gamma_P\rangle^2\right)\Bigg\}. \tag{16}$$

with $\Gamma_P = J_P' - \mathcal{F}_C^{-1}\mathcal{F}_C' J_P/2$.

Any operator not listed in Eqs. (15) and (16) and Table I is not renormalized at one-loop by scalar boson loops. In the SM limit one finds $\Omega = 0$, $\mathcal{K} = 2$ and $J_S'' = \Gamma_{S,P} = 0$, so all the $\mathcal{L}_4^{1\ell,\infty}$ operators in Eq. (16) and Table I vanish but for $\Gamma_{10}^{\rm SM} = -1/6$, which turns out to be independent of the Higgs field and is absorbed through the renormalization of g and g' in $\mathcal{L}_{YM}$. Furthermore, apart of $\mathcal{L}_{YM}$, only the non-derivative operators (the Yukawa term and Higgs potential) get renormalized due to the scalar loops in the SM limit.

RENORMALIZATION

In order to have a finite 1-loop effective action the divergences in Eq. (14) are canceled by the counterterms

$$\mathcal{L}^{\rm ct} = \sum_k c_k\,\mathcal{O}_k\,, \tag{17}$$

with the renormalization condition $c_k = c_k^r + \lambda\,\Gamma_k$.

The Γ_k's have a Taylor expansion of the form $\Gamma_k[h/v] = \sum_n \Gamma_{k,n}(h/v)^n/n!$, and similarly, $c_k[h/v] = \sum_n c_{k,n}(h/v)^n/n!$. The couplings of $\mathcal{O}(p^4)$ operators not present in Eqs. (15) and (16) do not get renormalized by scalar loops. This leads to the renormalization group equations for the $\mathcal{O}(p^4)$ coefficients,

$$\frac{\partial c_{k,n}^r}{\partial\ln\mu} = -\frac{\Gamma_{k,n}}{16\pi^2}\,. \tag{18}$$

Physically, this means that the NLO effective couplings will appear in the amplitudes in combinations with log-

arithms of energy scales E in the form

$$\mathcal{M}_{\mathcal{O}(p^4)} \propto \left(c^r_{k,n}(\mu) - \frac{\Gamma_{k,n}}{16\pi^2} \ln \frac{E}{\mu} \right) E^4 . \qquad (19)$$

As it is well known from Chiral Perturbation Theory, the size of these logs is not known a priori and can even be more dominant than the $\mathcal{O}(p^4)$ finite pieces. For instance, in QCD for $\mu \sim M_\rho$, the logarithms in the pion vector form-factor are numerically subdominant in comparison with the tree-level contributions, whereas one finds the opposite behavior in $\pi\pi$ production in the scalar-isoscalar channel. There is a priori no reason to neglect the running of the coefficients in the ECLh Lagrangian when confronting the experimental data against the (nonlinear) effective description of EWSB.

In Table I we have given the Γ_k that provide the full renormalization of the purely bosonic $\mathcal{L}_4$ Lagrangian for an arbitrary number of Higgs fields. In the last column one can find their contributions with the minimal number of Higgs fields, $\Gamma_{k,0} = \Gamma_k[0]$, which we have used as a check of our results. In the $\mathcal{F}_C = 1$ limit ($a = b = 0$) we recover the running of the Higgsless EW chiral Lagrangian [11, 12]. Part of our results for the 1-loop running have been already determined in the general ECLh case in WW, ZZ, hh-scattering ($c_{4,0}$,..., $c_{8,0}$) [19] and $\gamma\gamma$-scattering and related photon processes ($c_{1,0}$, $c_{2,0} - c_{3,0}$) [8]. Although the corresponding experimental analyses are limited so far by statistics and yield very loose constraints on these couplings [27–29], their accurate determination or the feasibility to set more stringent bounds in the future requires a careful control of these $\mathcal{O}(p^4)$ loop corrections.

Since the operators proportional to $A_{\mu\nu}A^{\mu\nu}$ and $A_{\mu\nu}Z^{\mu\nu}$ are only contained in the combination $\Delta\mathcal{L} = (c_1/4 + c_{10}/2)\langle f_+^{\mu\nu} f_{+\,\mu\nu} \rangle \subset \mathcal{L}_4$, the NLO couplings that describe the vertices $h \to \gamma\gamma$ and $h \to \gamma Z$ are renormalization group invariant, as was already found in [8] and [16], respectively. As $\Gamma_1/4 + \Gamma_{10}/2 = -1/12$ is independent of h, a similar thing applies to $\gamma\gamma$ and γZ vertices with more Higgs fields ($\gamma\gamma$, $\gamma Z \to hh$, hhh ...).

Deviations from the SM at LO (e.g., by having a value $a \neq 1$ in $\mathcal{F}_C$, that modifies the hWW coupling) would also imply the appearance of the four-fermion UV divergences in Eq. (16), and thus the contribution of the associated chiral logarithms to flavor-changing neutral current processes. In addition to the usual bounds on the corresponding four-fermion couplings, the study of these transitions may set strong constraints on the LO couplings in nonlinear Higgs EFT.

Let us finally observe that the Higgs potential gets divergent corrections proportional to $(V')^2$ and $(V'')^2$, *i.e.* proportional to m_h^4, that could be relevant in the study of the stability of the Higgs potential, a subject beyond the scope of this letter.

This article has been focused on the 1-loop contributions of SM scalar particles and the induced renormalization at NLO in the chiral counting. Because scalars couple derivatively in nonlinear EW models, the scalar loops are the only source of NLO divergences that scale like the fourth power of the external momenta, $(q_i)^4$, e.g. c_8 in Table I. The fermionic operators in $\mathcal{L}_4^{1\ell,\infty}|_{\rm ferm}$ in (16) scale as $(q_i)^3$ and $(q_i)^2$ (recall that the fermion field scales as the square root of the external momenta [7, 9, 10]), with the remaining powers of p given by the fermion masses in our computation. An analogous thing happens with the $\mathcal{L}_2$ renormalization in Eq. (15): the operators therein formally scale like $\mathcal{O}(p^4)$ but, since they are proportional to V' or V'', at least two of the powers of p are actually m_h^2. On the other hand, contributions from gauge bosons and fermions inside the loop, which are not included in this work, will produce UV divergences of order $(q_i)^3$ and $(q_i)^2$, as these particles couple non-derivatively and proportionally to the g and g' gauge couplings and Yukawas $y_{t,b}$ (*i.e.*, proportionally to the gauge boson and fermion masses.) The computation and analysis of the latter is postponed to future work.

Acknowledgements:
We would like to thank J.F. Donoghue, M.J. Herrero and A. Pich for discussions on the heat-kernel expansion and EW chiral Lagrangians, and S. Saa for pointing out the simplification in the structure of the Higgs kinetic term. PRF thanks P. Ruiz-Torres for stimulating and cheerful discussions. The work of FKG is supported in part by the NSFC and DFG through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD"(NSFC Grant No. 11261130311) and NSFC (Grant No. 11165005), the work of JJSC is supported by ERDF funds from the European Commission [FPA2010-17747, FPA2013-44773-P, SEV-2012-0249, CSD2007-00042], and the research of PRF was supported by the Munich Institute for Astro- and Particle Physics (MIAPP) of the DFG cluster of excellence "Origin and Structure of the Universe.

[1] H. Georgi and D.B. Kaplan, Phys.Lett. B145 (1984) 216.
[2] C.G. Callan, Jr., S.R. Coleman, J. Wess and B. Zumino, Phys.Rev. 177 (1969) 2247; S.R. Coleman, J. Wess and B. Zumino, Phys.Rev. 177 (1969) 2239.
[3] R. Contino, [arXiv:1005.4269 [hep-ph]].
[4] J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142; Nucl. Phys. B **250** (1985) 465.
[5] E.E. Jenkins, A.V. Manohar and M. Trott, JHEP 1310 (2013) 087 [arXiv:1308.2627 [hep-ph]]; JHEP 1401 (2014) 035 [arXiv:1310.4838 [hep-ph]]; R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, JHEP 1404 (2014) 159 [arXiv:1312.2014 [hep-ph]].
[6] Steven Weinberg, Physica A**96** (1979) 327.
[7] H. Georgi and A. Manohar Nucl.Phys. B **234** (1984) 189.
[8] R.L. Delgado, A. Dobado, M.J. Herrero and J.J. Sanz-Cillero, JHEP **1407** (2014) 149 [arXiv:1404.2866 [hep-ph]].

[9] J. Hirn and J. Stern, Eur.Phys.J. C **34** (2004) 447 [arXiv:hep-ph/0401032]; JHEP **0409** (2004) 058 [arXiv:hep-ph/0403017].

[10] G. Buchalla and O. Catà, JHEP **1207** (2012) 101 [arXiv:1203.6510 [hep-ph]]; G. Buchalla, O. Catà and C. Krause, Phys.Lett. B **731** (2014) 80 [arXiv:1312.5624 [hep-ph]]; Nucl.Phys. B **880** (2014) 552 [arXiv:1307.5017 [hep-ph]].

[11] A.C. Longhitano, Phys. Rev. D **22** (1980) 1166; Nucl. Phys. B **188** (1981) 118; T. Appelquist and C. Bernard, Phys.Rev. D **22** (1980) 200.

[12] M.J. Herrero and E. Ruiz-Morales, Nucl.Phys. B**418** (1994) 431 [arXiv:hep-ph/9308276]; **437** (1995) 319 [arXiv:hep-ph/9411207].

[13] M.B. Gavela, K. Kanshin, P.A.N. Machado and S. Saa, JHEP **1503** (2015) 043 [arXiv:1409.1571 [hep-ph]].

[14] CMS Coll., CMS-NOTE-13-002 [arXiv:1307.7135]; H. Baer *et al.*, ILC-REPORT-2013-040 [arXiv:1306.6352 [hep-ph]]; IHEP-CEPC-DR-2015-01 [http://cepc.ihep.ac.cn/preCDR/volume.html].

[15] R. Alonso, M.B. Gavela, L. Merlo, S. Rigolin and J. Yepes, Phys.Lett. B **722** (2013) 330; 726 (2013) 926 [arXiv:1212.3305 [hep-ph]].

[16] A. Azatov, R. Contino, A. Di Iura and J. Galloway, Phys.Rev. D **88** (2013) 7, 075019 [arXiv:1308.2676 [hep-ph]].

[17] A. Azatov, C. Grojean, A. Paul and E. Salvioni, Zh.Eksp.Teor.Fiz. **147** (2015) 410, J.Exp.Theor.Phys. **120** (2015) 354 [arXiv:1406.6338 [hep-ph]].

[18] A. Azatov, R. Contino, G. Panico and M. Son, [arXiv:1502.00539 [hep-ph]].

[19] R.L. Delgado, A. Dobado, F.J. Llanes-Estrada, JHEP **1402** (2014) 121 [arXiv:1311.5993 [hep-ph]]; D. Espriu, F. Mescia and B. Yencho, Phys.Rev. D **88** (2013) 055002 [arXiv:1307.2400 [hep-ph]].

[20] R. Contino, C. Grojean, M. Moretti, F. Piccinini, and R. Rattazzi, JHEP **1005** (2010) 089 [arXiv:1002.1011]; A. Azatov, R. Contino, and J. Galloway, JHEP **1204** (2012) 127; **1304** (2013) 140 [arXiv:1202.3415].

[21] G.F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP **0706** (2007) 045 [arXiv:hep-ph/0703164].

[22] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, Nucl.Phys. B **645** (2002) 155 [arXiv:hep-ph/0207036].

[23] G. 't Hooft, Nucl.Phys. B **62** (1973) 444; P. Ramond, Front.Phys. **74** (1989) 1; B.S. DeWitt, Int.Ser.Monogr.Phys. **114** (2003) 1; A.O. Barvinsky and G.A. Vilkovisky, Phys.Rept. **119** (1985) 1; C.Lee, T. Lee and H. Min, Phys.Rev. D **39** (1989) 1681; R.D. Ball, Phys.Rept. **182** (1989) 1; D.V. Vassilevich, Phys. Rept. **388** (2003) 279 [arXiv:hep-th/0306138].

[24] A. Pich, I. Rosell and J.J. Sanz-Cillero, Phys.Rev.Lett. **110** (2013) 181801 [arXiv:1212.6769]; JHEP **1401** (2014) 157 [arXiv:1310.3121 [hep-ph]].

[25] I. Rosell, P. Ruiz-Femenia and J. Portoles, JHEP **0512** (2005) 020 [arXiv:hep-ph/0510041].

[26] J.F. Donoghue, E. Golowich and B.R. Holstein, *Dynamics of the standard model*, Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 2 (1992) 1-540.

[27] ATLAS Collaboration, Report No. ATLAS-CONF-2014-009; CMS Collaboration, Report No. CMS-PAS-HIG-14-009.

[28] G. Aad et al. (ATLAS Collaboration), Phys.Rev.Lett. **113** (2014) 14, 141803 [arXiv:1405.6241 [hep-ex]].

[29] S. Chatrchyan et al. (CMS Collaboration), JHEP **1307** (2013) 116 [arXiv:1305.5596 [hep-ex]].